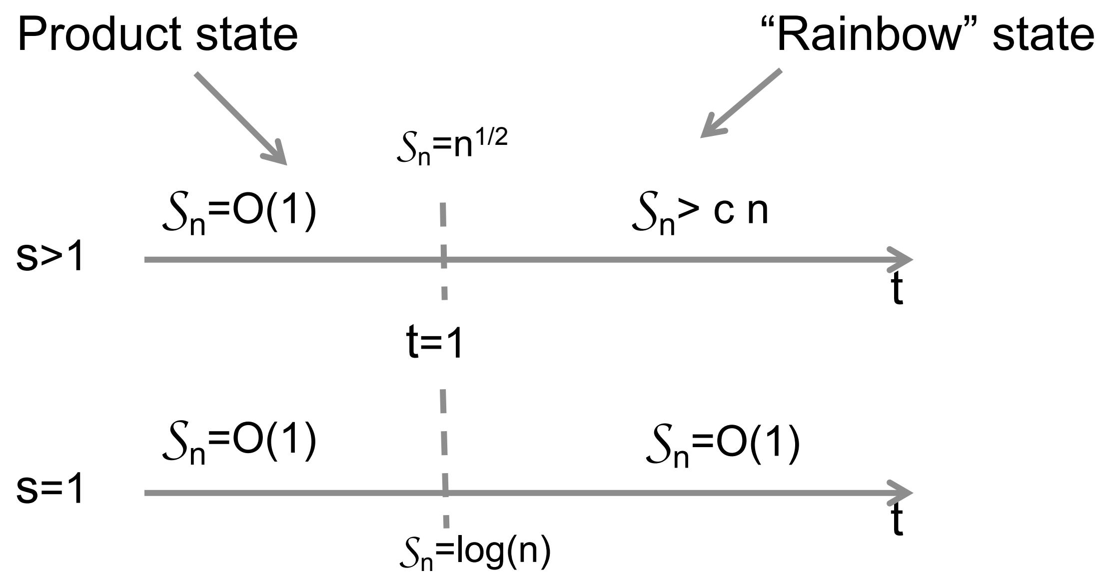
A QUANTUM PHASE TRANSITION FROM BOUNDED TO EXTENSIVE ENTANGLEMENT



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arXiv:1606.07795

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HIGHLY ENTANGLED STATES

Entanglement entropy:

$$S_A = -Tr\rho_A \log \rho_A \text{ where } \rho_A = Tr_B \rho$$

Generic states in Hilbert space have extensive entanglement

(page prl 93,foong prl 94,sen prl 96)

$$S_A \approx \begin{cases} L^d & \textit{generic state} \\ L^{d-1} & \textit{gapped, "area law"} \\ L^{d-1}\log L & \textit{free fermions} \\ \frac{c}{3}\log L & \textit{conformal} \end{cases}$$

generic state	(Page prl 93)
gapped, "area law"	(Hastings 07,1d)
free fermions	(Gioev IK 06,M Wolf 06)
conformal	(Holzhey Larsen Wilczek 96, Calabrese Cardy…)

EXTENSIVELY ENTANGLED STATES

First local Hamiltonian with volume scaling: Irani 2010.

local Hilbert space dimension is 21

Simpler models but without translational invariance, and with exponentially varying couplings:

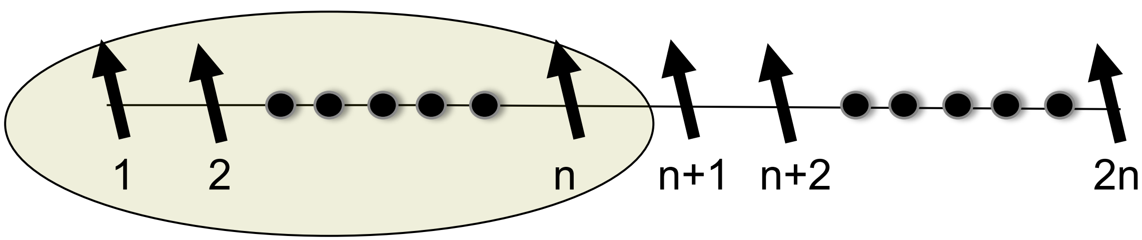
Gottesman Hastings 2010

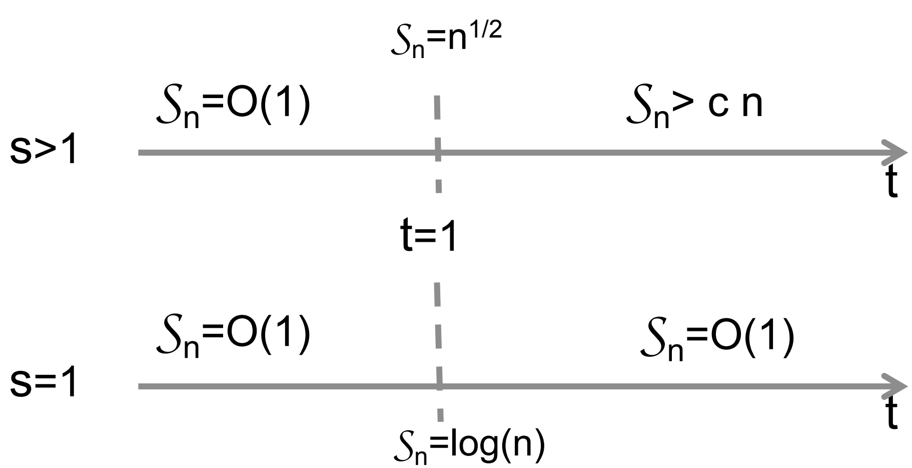
Rainbow ground states:Vitagliano Riera Latorre 2010, Ramirez Rodriguez-Laguna Sierra 2014

Translationally invariant but with a square root scaling:

Movassagh Shor (2014), Salberger Korepin (2016)

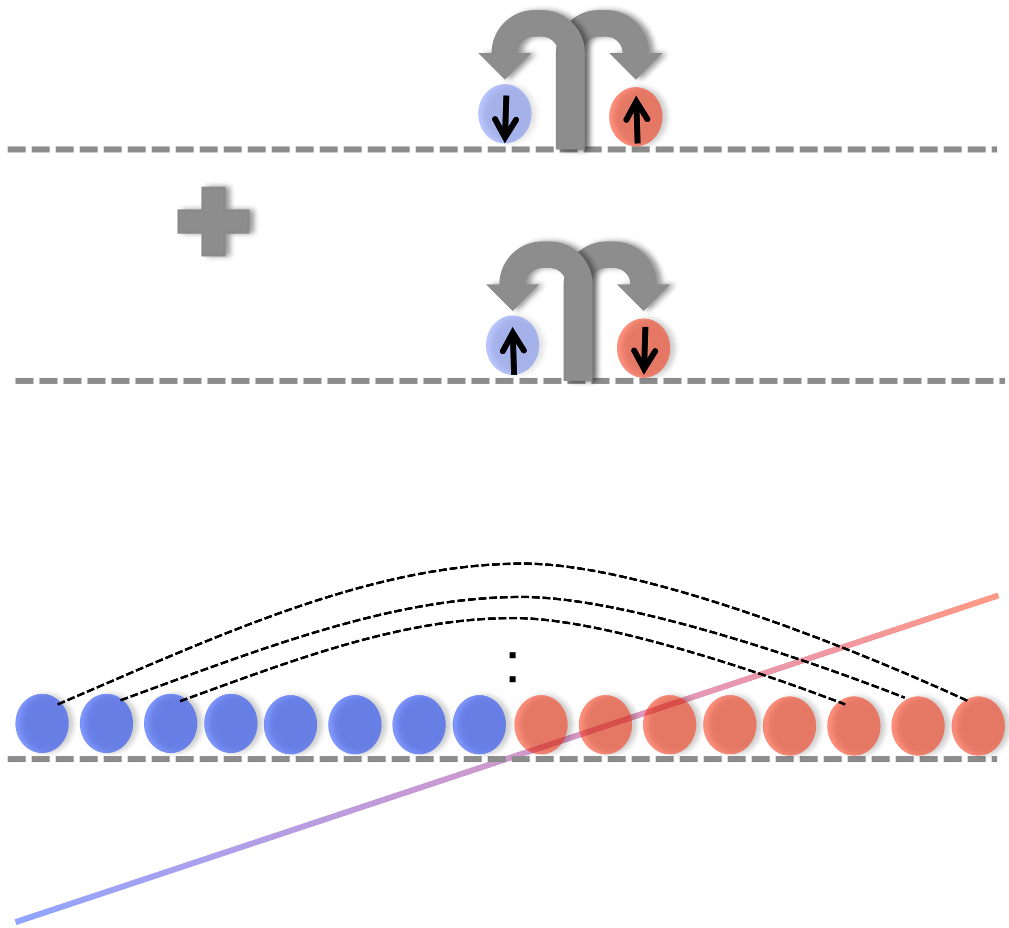
Here: a simple spin chain with remarkable phase transition:





Basic intuition: How to create a highly entangled state?

EPR: electron-positron pair generation in an electric field as a source of entanglement



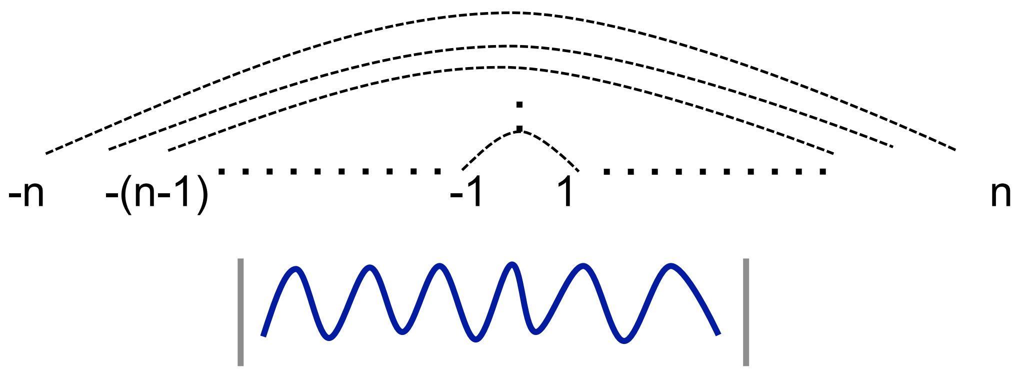
“Rainbow” state

ANOTHER TYPE OF RAINBOW STATE IN THE LAB!

Pfister et al, 2004
Chen Meniccuci Pfister PRL2014, 60 mode cluster state

Optical frequency comb

Cavity eigenmodes



Nonlinear cavity

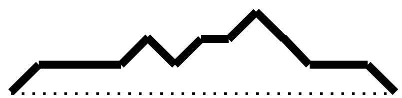
$$\omega_{in} \rightarrow \omega_n + \omega_{-n} = \omega_{in}$$

Incoming laser

MOTZKIN WALK HAMILTONIANS

Bravyi et al. 2012 “Criticality without frustration”

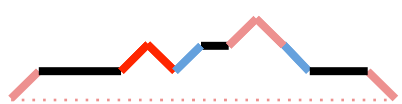
$$|\Psi\rangle = \sum_{\substack{\textit{Motzkin}\\ \textit{paths}}} \Big| \quad \Big\rangle$$



$$S_n \propto \frac{1}{2}\log(n)$$

Movassagh Shor 2014 “Power law violation of the area law in quantum spin chains”

$$|\Psi\rangle = \sum_{\substack{\textit{colored}\\ \textit{Motzkin}\\ \textit{paths}}} \Big| \quad \Big\rangle$$



$$S_n \propto \sqrt{n}$$

REPRESENTING SPIN STATES AS MOTZKIN WALKS

Motzkin paths: |1 , 0 , -1 , 1 , 1 , -1 , 1 , 0 , 1 , -1 , -1 , 0 , 0 , -1 >

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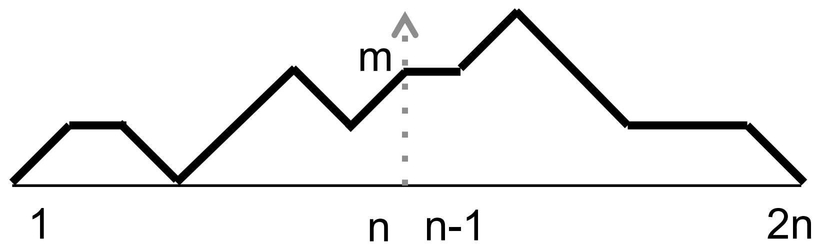

Colored Motzkin paths: |1 , 0, -1 , 2 , 1 ,-1 , 1 , 0, 2 , -2, -1 , 0, 0, -1>

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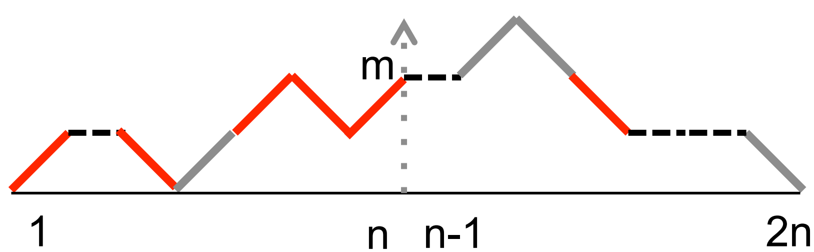
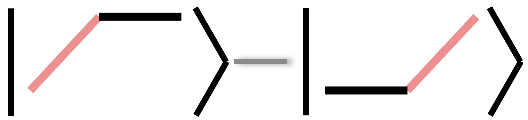
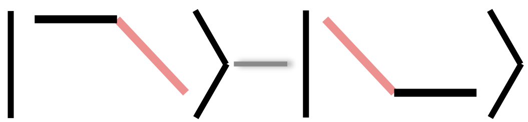
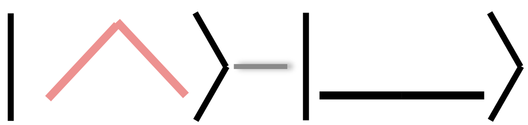


MOTZKIN HAMILTONIANS

Enforce a g.s. superposition made of Motzkin paths by using projectors like:

$|\Phi\rangle =$ 

$|\Psi\rangle =$ 

$|\Theta\rangle =$ 

$$H = \sum |\Theta\rangle\langle\Theta| + |\Psi\rangle\langle\Psi| + |\Phi\rangle\langle\Phi| +$$
$$h_1 + h_{2n} + (penalty\ \ unmatched\ \ colors)$$

HOW COLOR ENHANCES ENTROPY

$\text{Height after } n \text{ steps} = \# \text{ of unmatched up steps}$

$\text{For } n >> 1, \text{ typical Motzkin walk is like a Brownian walk.}$

$\Rightarrow$

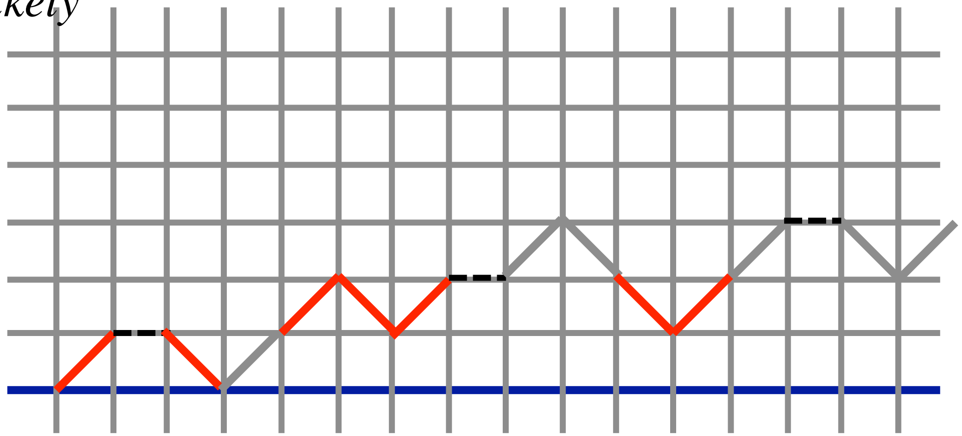
$\text{Typical height after } n \text{ steps} \propto \sqrt{n}$

$\Rightarrow$

$\# \text{ of colorings of unmatched up steps} \propto s^{\sqrt{n}}$

$\text{all coloring schemes of unmatched equally likely}$

$\Rightarrow \; S_n \propto \sqrt{n}$



CAN WE SKEW THE MODEL TO PREFER HIGHER MOTZKIN PATHS?

Main idea – up moves are like electrons and down moves are like positrons. They should go in different directions!

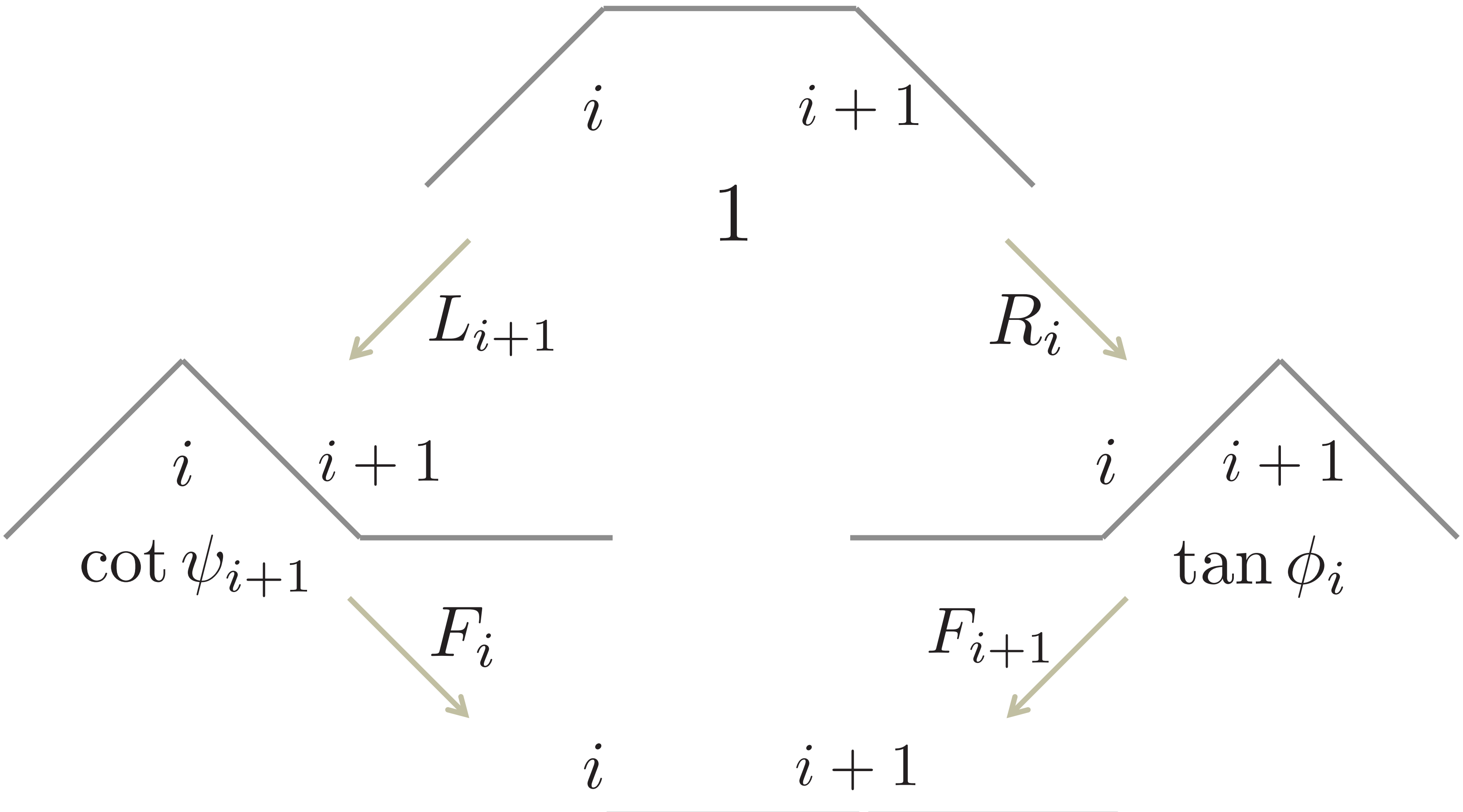
Can try:

$$|\Phi\rangle = \cos\varphi_i \left|\diagup\overline{\quad}\right\rangle - \sin\varphi_i \left|\underline{\quad}\diagup\right\rangle$$

$$|\Psi\rangle = \cos\psi_i \left|\overline{\quad}\diagdown\right\rangle - \sin\psi_i \left|\diagdown\underline{\quad}\right\rangle$$

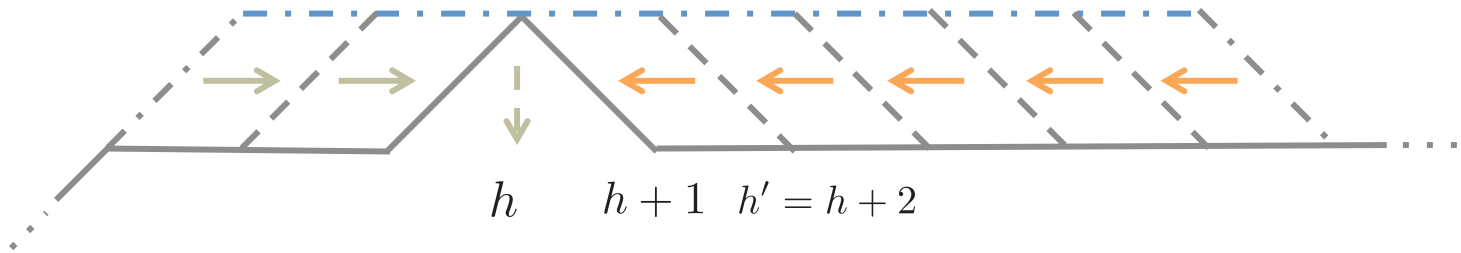
$$|\Theta\rangle = \cos\theta_i \left|\diagup\diagdown\right\rangle - \sin\theta_i \left|\underline{\quad}\,\underline{\quad}\right\rangle$$

Choice of angles must satisfy a consistency condition:

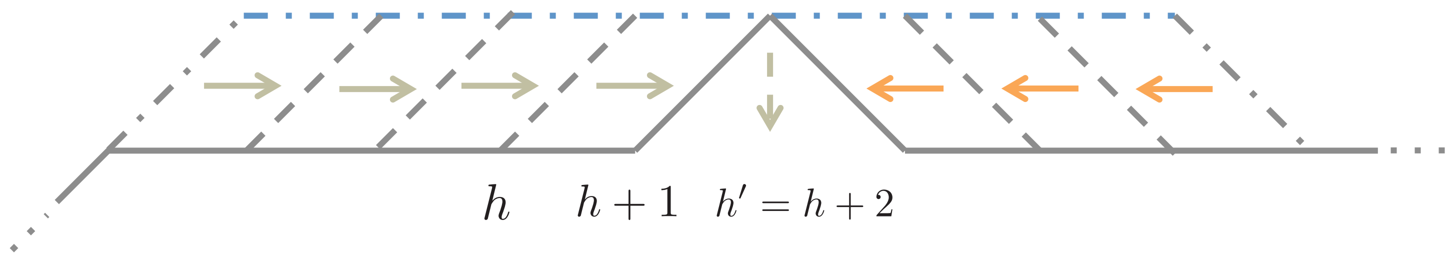


$$\cot\psi_{i+1}\tan\theta_i \equiv \tan\phi_i \tan\theta_{i+1}$$

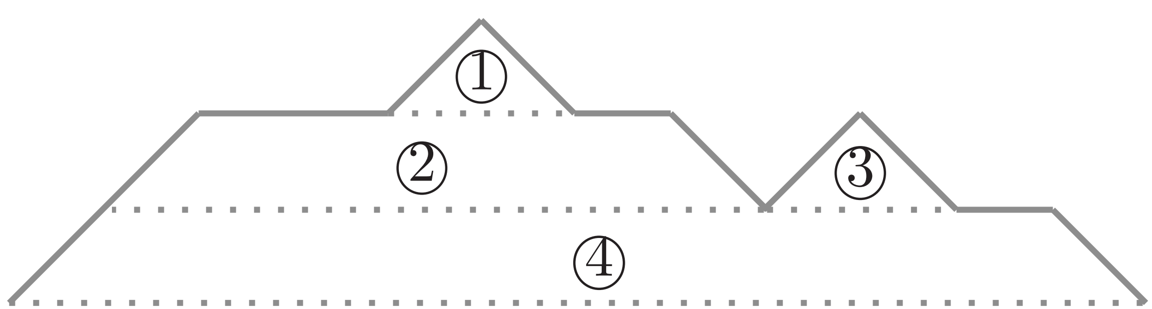
Local consistency condition is enough



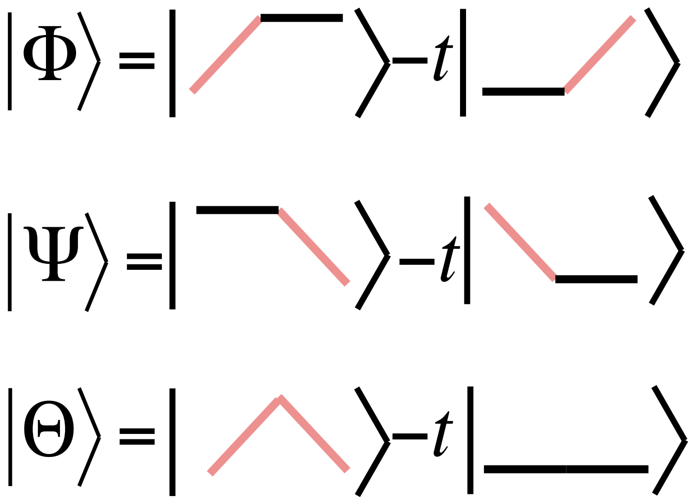
(a)

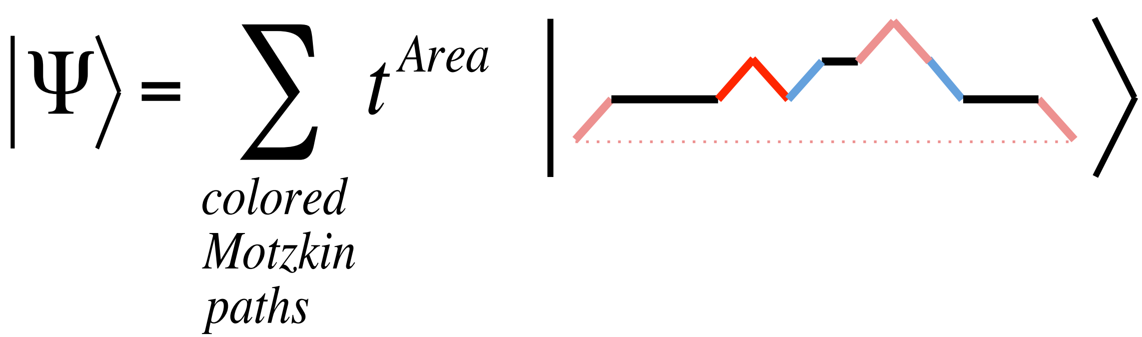


(b)



THE UNIFORM MODEL

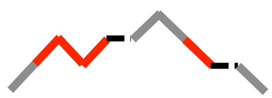




ENTANGLEMENT ENTROPY

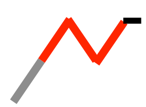
Schmidt decomposition

$$|\Psi\rangle = \sum_{\substack{colored \\ Motzkin \\ paths}} t^{Area} \, \Big|$$

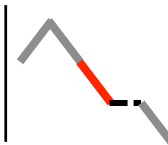


$$\Big\rangle$$

$$|\Psi\rangle = \sum_{m=0}^{n} \sqrt{p_{n,m}} \sum_{\substack{coloring \\ scheme}} \left(\sum_{\substack{paths\ \ from\ \ 0 \\ to\ \ height\ \ m}} t^{Area} \Big|\right.$$



$$\left.\Big\rangle \right) \otimes \left(\sum_{\substack{paths\ \ from \\ height\ \ m\ \ to\ \ 0}} t^{Area} \Big| \right.$$



$$\left.\Big\rangle \right)$$

$$p_{n,m} = \frac{M_{n,m}^2}{N_n}$$

$$M_{n,m} = \sum_{i=0}^{(n-m)/2} s^i \sum_{\substack{path\ from\ 0\ to \\ height\ m\ with \\ i\ unpaired\ colors}} t^{Area\ under\ path}$$

$$N_n = \sum_{m=0}^{n} s^m M_{n,m}^2$$

SCALING OF ENTROPY.

We need the asymptotics of $M_{n,m}$

$$M_{n,m} = \sum_{i=0}^{(n-m)/2} s^i \sum_{\substack{\text{path from 0 to}\\ \text{height } m \text{ with}\\ i \text{ unpaired colors}}} t^{\text{Area under path}}$$

$$\sum_{\substack{\text{path from 0 to}\\ \text{height } m \text{ with}}} t^{\text{Area under path}} \approx \int_{X(0)=0}^{X(n)=m} dX[\tau]\, e^{-\int_0^n (\frac{dX}{ds})^2 - \log(t) X(s) ds}$$

Charged particle in a field,
Brownian particle with a drift

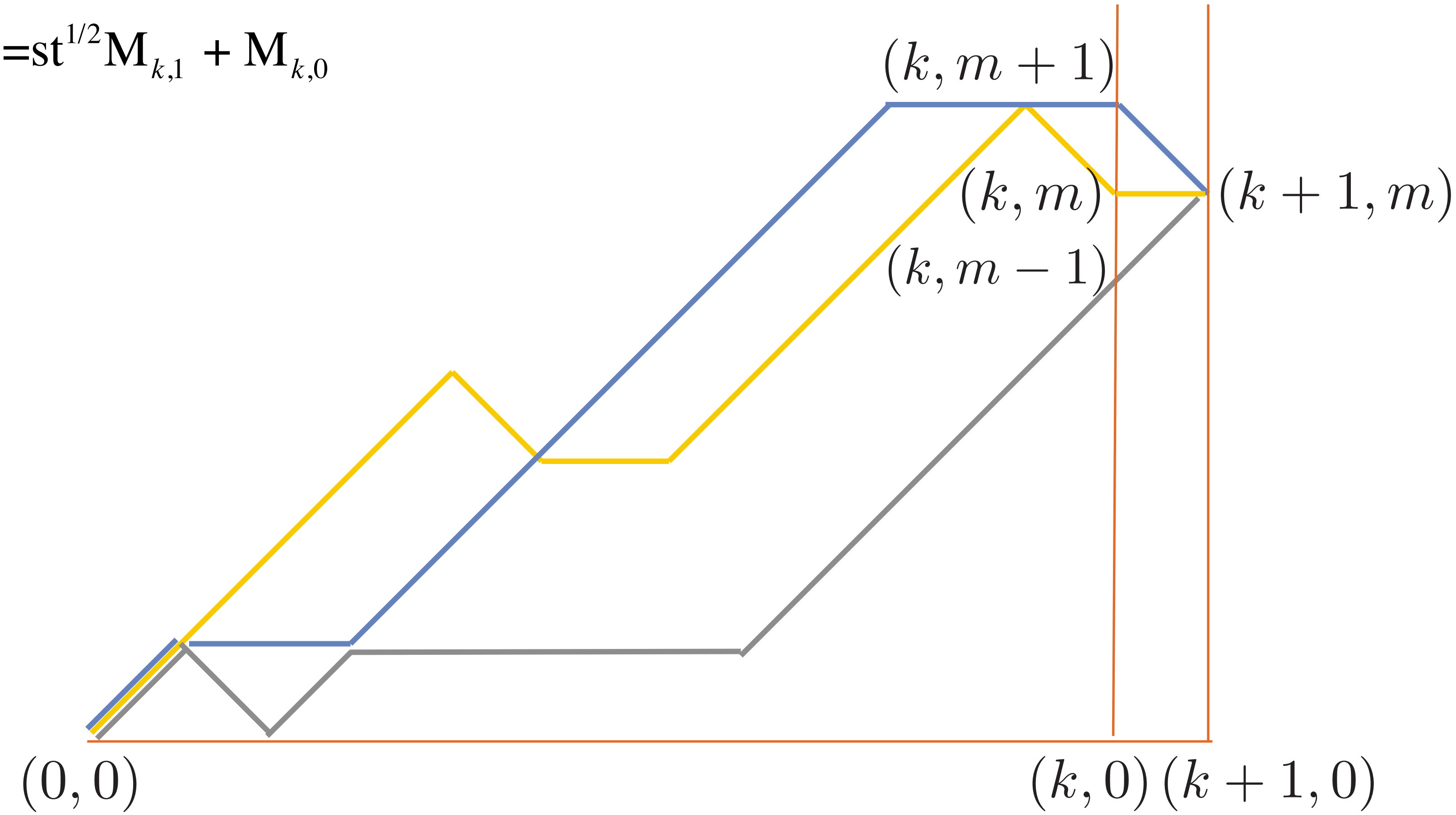
For precise estimates use recursion relations:

$$M_{k+1,k+1} = t^{k+1/2} M_{k,k}$$

$$M_{k+1,k} = t^{k} M_{k,k} + t^{k-1/2} M_{k,k-1}$$

$$M_{k+1,m} = s t^{m+1/2} M_{k,m+1} + t^{m} M_{k,m} + t^{m-1/2} M_{k,m-1}, \quad 0 < m < k$$

$$M_{k+1,0} = s t^{1/2} M_{k,1} + M_{k,0}$$



PROOF IDEA

Define

$$|M_n\rangle = \sum_{m=0}^{\infty} \mathrm{M}_{n,m}|m\rangle \quad ; \quad Shift: \ \hat{S}|m\rangle = |m-1\rangle$$

Then:

$$|M_n\rangle = \vec{K}\prod_{k=1}^{n}\left(st^{-(k-1/2)}\hat{S} + 1 + t^{(k-1/2)}\hat{S}^{+}\right)|0\rangle$$

For large n,

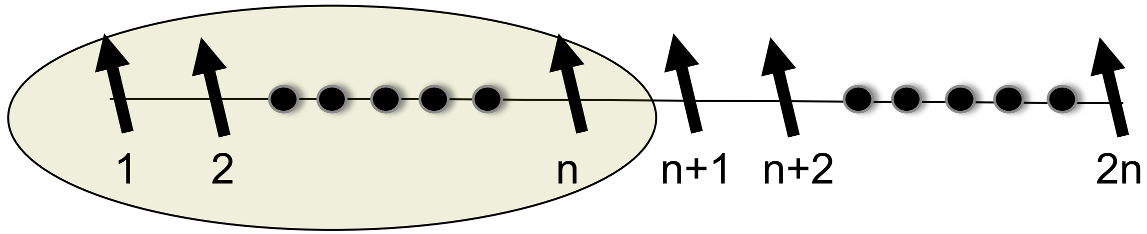
$$|M_n\rangle \sim \underbrace{\vec{K}\prod_{k=1}^{k_0-1}\left(st^{-(k-1/2)}\hat{S} + 1 + t^{(k-1/2)}\hat{S}^{+}\right)}_{\text{Transient}}\underbrace{\prod_{k=k_0}^{n}\left(t^{(k-1/2)}\hat{S}^{+}\right)|0\rangle}_{\propto |n-k_0\rangle} + corrections$$

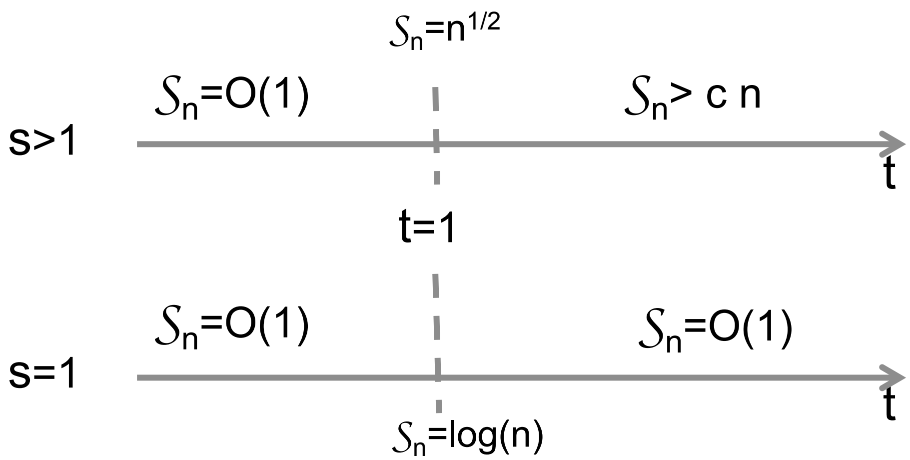
Transient

$\propto |n-k_0\rangle$

Ballistic propagation of distribution

Here: a simple spin chain with remarkable phase transition:

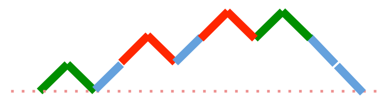




DEFORMED FREDKIN MODEL

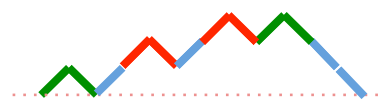
Fredkin model Salberger/Korepin 2016 has as ground state superposition of Dyck paths:

$$|\Psi\rangle = \sum_{\substack{colored\\ Dyck\\ paths}} \Big| \quad \Big\rangle$$



We can deform it into:

$$|\Psi\rangle = \sum_{\substack{colored\\ Dyck\\ paths}} q^{Area\ \ under} \Big| \quad \Big\rangle$$



Entropy scales linearly with n log(s)! Same phase diagram.
Need 3 neighbor interactions.
To appear shortly!
IK with Z Zhang, O Salberger, T Udagawa, H Katsura, V Korepin

ODDS AND ENDS

1. Gap decays exponentially for $t>1$. Gapped for $t<1$?
2. Thermodynamics is unknown (Shape of transition region?)
3. Stability?
4. Periodic boundary conditions?
5. Can build a tensor network.
6. Holography: Can get linear entanglement scaling by choosing a metric that would give entanglement using Ryu Takayangi formula. Relation to hyperscaling violations (Huijse, Sachdev and Swingle 2012)?